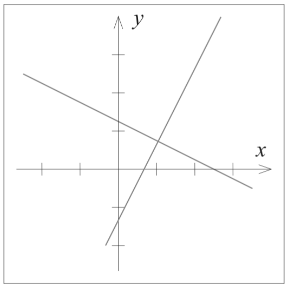
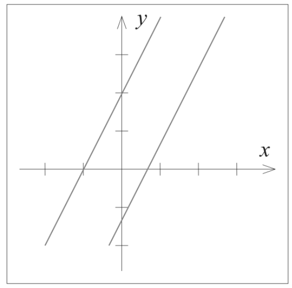
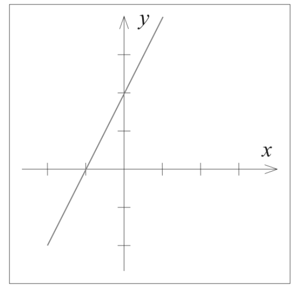
SOLUTIONS - CHAPTER 3 EXERCISES

3.1 One Solution | No Solution | Infinite Solutions







3.3 By Substitution Solve the second equation for x:

$$\begin{aligned} -x - \tfrac{2}{3}y &= 2 \\ -x &= \tfrac{2}{3}y + 2 \\ x &= \tfrac{-2}{3}y - 2 \end{aligned}$$

Substitute the result into the first equation:

$$\begin{aligned} 3(\tfrac{-2}{3}y - 2) + 2y &= -6 \\ -2y - 6 + 2y &= -6 \\ -6 &= -6 \end{aligned}$$

This statement is always true and tells us this system has infinitely many solutions. The solutions are of the form $(x, y) = (\tfrac{-2}{3}y - 2, y)$, where y is any real number.

By Matrices First form the augmented matrix:

$$\left[\begin{array}{cc|c} 3 & 2 & -6 \\ -1 & \tfrac{-2}{3} & 2 \end{array}\right]$$

Begin performing row operations. Multiply Row 1 by $\tfrac{1}{3}$ to obtain our first leading 1.

$$\begin{array}{r} \tfrac{1}{3}R_1 \to R_1 \\ R_2 \end{array} \left[\begin{array}{cc|c} 1 & \tfrac{2}{3} & -2 \\ -1 & \tfrac{-2}{3} & 2 \end{array}\right]$$

Add Row 1 to Row 2 to "zero-out" the first column.

$$\begin{array}{r} R_1 \\ R_1 + R_2 \to R_2 \end{array} \left[\begin{array}{cc|c} 1 & \tfrac{2}{3} & -2 \\ 0 & 0 & 0 \end{array}\right]$$

There are no more reductions possible so we have the system

$$x + \tfrac{2}{3}y = -2$$
$$0 = 0$$

Solving the top equation for x gives

$$x = -2 - \tfrac{2}{3}y$$

and the infinitely many solutions are of the form

$$(x, y) = (-2 - \tfrac{2}{3}y, y)\text{, where } y \text{ is any real number.}$$

3.5 Multiply each entry in the first matrix by 2:

$$\begin{bmatrix} 6 & 2a \\ 0 & -2 \end{bmatrix} - \begin{bmatrix} b & 2 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ c & d \end{bmatrix}$$

Subtract the matrices on the left:

$$\begin{bmatrix} 6-b & 2a-2 \\ 0-(-2) & -2-6 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ c & d \end{bmatrix}$$

Simplify:

$$\begin{bmatrix} 6-b & 2a-2 \\ 2 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ c & d \end{bmatrix}$$

Set corresponding entries equal to each other and solve for the different variables:

$$\begin{aligned} 6-b &= 1 \\ -b &= -5 \\ b &= 5 \end{aligned} \qquad \begin{aligned} 2a-2 &= 5 \\ 2a &= 7 \\ a &= \tfrac{7}{2} \end{aligned} \qquad 2 = c \qquad -8 = d$$

Thus, $a = \tfrac{7}{2}, b = 5, c = 2,$ and $d = -8$.

3.7 a. True
b. False → Singular matrices do not have inverses.
c. True
d. True

3.9 Gauss-Jordan Method First form the augmented matrix:

$$\left[\begin{array}{ccc|c} 3 & 1 & -2 & 5 \\ 2 & 1 & -1 & 3 \\ 4 & 2 & -1 & 6 \end{array}\right]$$

Multiply Row 1 by $\frac{1}{3}$ to obtain a leading 1:

$$\begin{array}{r} \frac{1}{3}R_1 \to R_1 \\ R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|c} 1 & \frac{1}{3} & -\frac{2}{3} & \frac{5}{3} \\ 2 & 1 & -1 & 3 \\ 4 & 2 & -1 & 6 \end{array}\right]$$

Multiply Row 1 by -2 and add the result to Row 2:

$$\begin{array}{r} R_1 \\ -2R_1 + R_2 \to R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|c} 1 & \frac{1}{3} & -\frac{2}{3} & \frac{5}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\ 4 & 2 & -1 & 6 \end{array}\right]$$

Multiply Row 1 by -4 and add the result to Row 3:

$$\begin{array}{r} R_1 \\ R_2 \\ -4R_1 + R_3 \to R_3 \end{array} \left[\begin{array}{ccc|c} 1 & \frac{1}{3} & -\frac{2}{3} & \frac{5}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{2}{3} & \frac{5}{3} & -\frac{2}{3} \end{array}\right]$$

Multiply Row 2 by 3 to obtain a leading 1:

$$\begin{array}{r} R_1 \\ 3R_2 \to R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|c} 1 & \frac{1}{3} & -\frac{2}{3} & \frac{5}{3} \\ 0 & 1 & 1 & -1 \\ 0 & \frac{2}{3} & \frac{5}{3} & -\frac{2}{3} \end{array}\right]$$

"Zero-out" the second column by the following operations –

- Multiply Row 2 by $-\frac{1}{3}$ and add the result to Row 1.
- Multiply Row 2 by $-\frac{2}{3}$ and add the result to Row 3.

$$\begin{array}{r} -\frac{1}{3}R_2 + R_1 \to R_1 \\ R_2 \\ -\frac{2}{3}R_2 + R_3 \to R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

"Zero-out" the third column by the following operations –

- Add Row 3 to Row 1.
- Multiply Row 3 by -1 and add the result to Row 2.

$$\begin{array}{r} R_3 + R_1 \to R_1 \\ -1R_3 + R_2 \to R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

This results in the system

$$\begin{aligned} x &= 2 \\ y &= -1 \\ z &= 0 \end{aligned}$$

which is the solution to our original system.

Inverse Method Let $\mathbf{A} = \begin{bmatrix} 3 & 1 & -2 \\ 2 & 1 & -1 \\ 4 & 2 & -1 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 5 \\ 3 \\ 6 \end{bmatrix}$, and $\mathbf{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ so that **AX=B** gives us our system. Using a calculator or other tool, find that

$$\mathbf{A}^{-1} = \begin{bmatrix} 1 & -3 & 1 \\ -2 & 5 & -1 \\ 0 & -2 & 1 \end{bmatrix}$$

Solve for **X** by finding the product $\mathbf{A}^{-1} \cdot \mathbf{B}$:

$$\mathbf{X} = \mathbf{A}^{-1} \cdot \mathbf{B} = \begin{bmatrix} 1 & -3 & 1 \\ -2 & 5 & -1 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$

Thus, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} \Rightarrow x = 2, y = -1, z = 0$ is our solution.