SOLUTIONS - CHAPTER 3 QUIZ

3.1 a. C → Parallel lines will not cross, so they have no points in common.
b. A → Two intersecting lines will meet at only one point.
c. B → Two lines on top of each other have all points (infinitely many) in common.

3.2 By Substitution Use one of the equations to solve for one variable; here, we choose to solve the first equation for *y*:

$$\begin{aligned} -5x-2y &= 20 \\ -2y &= 5x+20 \\ y &= \tfrac{-5}{2}x-10 \end{aligned}$$

Substitute the result into the second equation:

$$\begin{aligned} 10x+4(\tfrac{-5}{2}x-10) &= 10 \\ 10x-10x-10 &= 10 \\ -10 &= 10 \end{aligned}$$

This result is never true and tells us that the original system has NO SOLUTION.

By Matrices Form the augmented matrix:

$$\left[\begin{array}{cc|c} -5 & -2 & 20 \\ 10 & 4 & 10 \end{array}\right]$$

Begin performing row operations. Multiply Row 1 by $\tfrac{-1}{5}$ to obtain our first leading 1.

$$\begin{array}{c} \tfrac{1}{5}R_1 \rightarrow R_1 \\ R_2 \end{array} \left[\begin{array}{cc|c} 1 & \tfrac{2}{5} & -4 \\ 10 & 4 & 10 \end{array}\right]$$

"Zero-out" the first column by multiplying Row 1 by -10 and adding the result to Row 2.

$$\begin{array}{c} R \\ -10R_1+R_2 \rightarrow R_2 \end{array} \left[\begin{array}{cc|c} 1 & \tfrac{2}{5} & -4 \\ 0 & 0 & 50 \end{array}\right]$$

The last row of this matrix says that $0=50$. This statement is never true and tells us that the original system has NO SOLUTION.

3.3 By Substitution Substitute the second equation into the first equation:

$$\begin{aligned} 3x-4(4)&=12\\ 3x-16&=12\\ 3x&=28\\ x&={}^{28}\!/_{3} \end{aligned}$$

Thus, the solution is $(x,y)=({}^{28}\!/_{3},4)$.

3.4 By Substitution Because there is no easy variable to solve for in either equation, choose either one. Here, we choose to solve for y in the first equation:

$$\begin{aligned} -4x+2y&=8\\ 2y&=4x+8\\ y&=2x+4 \end{aligned}$$

Substitute the result into the second equation:

$$\begin{aligned} 8x-4(2x+4)&=-16\\ 8x-8x-16&=-16\\ -16&=-16 \end{aligned}$$

This statement is always true, so there are infinitely many solutions. The solutions are of the form $(x,y)=(x,2x+4)$, where x is any real number.

By Matrices First, form the augmented matrix:

$$\left[\begin{array}{cc|c} -4 & 2 & 8\\ 8 & -4 & -16 \end{array}\right]$$

Begin performing row operations. Multiply Row 1 by ${}^{-1}\!/_{4}$ to form the first leading 1.

$$\begin{array}{r} {}^{-1}\!/_{4}R_1\to R_1\\ R_2 \end{array}\left[\begin{array}{cc|c} 1 & {}^{-1}\!/_{2} & -2\\ 8 & -4 & -16 \end{array}\right]$$

“Zero-out” the first column by multiplying Row 1 by -8 and adding the result to Row 2.

$$\begin{array}{r} R_1\\ -8R_1+R_2\to R_2 \end{array}\left[\begin{array}{cc|c} 1 & {}^{-1}\!/_{2} & -2\\ 0 & 0 & 0 \end{array}\right]$$

No further simplification is possible and the resulting equivalent system is

$$\begin{aligned} x-{}^{1}\!/_{2}\,y&=-2\\ 0&=0 \end{aligned}$$

Since there are more variables than equations containing variables, this means there will be infinitely many solutions. To find the form of these solutions, solve the remaining equation for one of the variables in terms of the other.

Thus, solving for *x* we have $x=-2+\frac{1}{2}y$, giving solutions of the form

$$(x,y)=(-2+\tfrac{1}{2}y,y)\text{, where } y \text{ is any real number}$$

or solving for *y* we have $y=4+2x$, giving solutions of the form

$$(x,y)=(x,4+2x)\text{, where } x \text{ is any real number}$$

3.5 First, multiply the entries in the second matrix by 3:

$$\begin{bmatrix} w & x \\ y & z \end{bmatrix}+\begin{bmatrix} -3 & 0 \\ 12 & 6 \end{bmatrix}=\begin{bmatrix} 5 & 3 \\ 1 & -3 \end{bmatrix}$$

Next, add the matrices on the left:

$$\begin{bmatrix} w-3 & x+0 \\ y+12 & z+6 \end{bmatrix}=\begin{bmatrix} 5 & 3 \\ 1 & -3 \end{bmatrix}$$

Set corresponding entries equal to each other and solve for the different variables:

$$\begin{array}{cccc} w-3=5 & x=3 & y+12=1 & z+6=-3 \\ w=8 & & y=-11 & z=-9 \end{array}$$

Thus, $w=8, x=3, y=-11,$ and $z=-9$.

3.6 When determining the dimensions of matrix products, it is easiest to write the dimensions of the matrices being multiplied (IN THE CORRECT ORDER), compare the inner dimensions, and use the outer dimensions for the product dimension if the inner dimensions are the same.

a. EF: (1 x 5)(5 x 1) ⇒ 1 x 1
b. FE: (5 x 1)(1 x 5) ⇒ 5 x 5
c. EG: (1 x 5)(5 x 5) ⇒ 1 x 5
d. GE: (5 x 5)(1 x 5) ⇒ Not possible – product does not exist
e. HG: (1 x 1)(5 x 5) ⇒ Not possible – product does not exist
f. GH: (5 x 5)(1 x 1) ⇒ Not possible – product does not exist
g. HF: (1 x 1)(5 x 1) ⇒ Not possible – product does not exist
h. FH: (5 x 1)(1 x 1) ⇒ 5 x 1

3.7 a. False → $\begin{bmatrix} 1 & -1 & 2.5 \\ 3 & -1 & 3.5 \\ -2 & 1 & -3 \end{bmatrix}$ is a square matrix and it does not have an inverse.

b. False → A requirement to having an inverse is that the matrix must be square.

c. True → Multiplying by the identity matrix is similar to multiplying a number by 1.

d. True → The inverse is a matrix that has the property that $\mathbf{A}\cdot\mathbf{A}^{-1}=\mathbf{A}^{-1}\cdot\mathbf{A}=\mathbf{I}$.

3.8 Conditions a and e must be met:

a → The coefficient matrix must be square to have an inverse which means the number of equations must equal the number of variables.

b,c → False due to the reason given for condition a above.

e → $\mathbf{A}^{-1}\cdot\mathbf{B}$ will result in a single matrix by virtue of matrix multiplication which means there will be exactly one solution.

d,f → False due to the reason given for condition e above.

3.9 Gauss-Jordan Method First form the augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & -5 \\ -1 & -4 & 1 & 6 \\ 2 & 6 & -1 & -8 \end{array}\right]$$

“Zero-out” Row 1 by the following operations —

- Add Row 1 to Row 2.
- Multiply Row 1 by -2 and add the result to Row 3.

$$\begin{array}{r} R_1 \\ R_1+R_2\rightarrow R_2 \\ -2R_1+R_3\rightarrow R_3 \end{array}\left[\begin{array}{ccc|c} 1 & 3 & -1 & -5 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

Multiply Row 2 by -1 to obtain a leading 1:

$$\begin{array}{r} R_1 \\ -1R_2\rightarrow R_2 \\ R_3 \end{array}\left[\begin{array}{ccc|c} 1 & 3 & -1 & -5 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

“Zero-out” the second column by multiplying Row 2 by -3 and adding the result to Row 1:

$$\begin{array}{r} -3R_2+R_1\rightarrow R_1 \\ R_2 \\ R_3 \end{array}\left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

“Zero-out” the third column by adding Row 3 to Row 1:

$$\begin{array}{r} R_3+R_1 \to R_1 \\ R_2 \\ R_3 \end{array}\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

The resulting system is

$$\begin{aligned} x &= 0 \\ y &= -1 \\ z &= 2 \end{aligned}$$

which is the solution to our original system.

Inverse Method Let $\mathbf{A}=\begin{bmatrix} 1 & 3 & -1 \\ -1 & -4 & 1 \\ 2 & 6 & -1 \end{bmatrix}$, $\mathbf{B}=\begin{bmatrix} -5 \\ 6 \\ -8 \end{bmatrix}$, and $\mathbf{X}=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ so that **AX=B** gives us our system. Using a calculator or other tool, find that

$$\mathbf{A}^{-1}=\begin{bmatrix} 2 & 3 & 1 \\ -1 & -1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

Solve for **X** by finding the product $\mathbf{A}^{-1}\cdot\mathbf{B}$:

$$\mathbf{X}=\mathbf{A}^{-1}\cdot\mathbf{B}=\begin{bmatrix} 2 & 3 & 1 \\ -1 & -1 & 0 \\ -2 & 0 & 1 \end{bmatrix}\begin{bmatrix} -5 \\ 6 \\ -8 \end{bmatrix}=\begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$$

Thus, $\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} \Rightarrow x=0, y=-1, z=2$ is our solution.

3.10 Gauss-Jordan Method First, form the augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & -1 & 2.5 & 3 \\ 3 & -1 & 3.5 & 7 \\ -2 & 1 & -3 & -5 \end{array}\right]$$

"Zero-out" the first column by the following operations –

- Multiply Row 1 by -3 and add the result to Row 2.
- Multiply Row 1 by 2 and add the result to Row 3.

$$\begin{array}{r} R_1 \\ -3R_1 + R_2 \to R_2 \\ 2R_1 + R_3 \to R_3 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 2.5 & 3 \\ 0 & 2 & -4 & -2 \\ 0 & -1 & 2 & 1 \end{array}\right]$$

Multiply Row 2 by ½ to obtain a leading 1:

$$\begin{array}{r} R_1 \\ \tfrac{1}{2}R_2 \to R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 2.5 & 3 \\ 0 & 1 & -2 & -1 \\ 0 & -1 & 2 & 1 \end{array}\right]$$

“Zero-out the second column by the following operations –

- Add Row 2 to Row 1.
- Add Row 2 to Row 3.

$$\begin{array}{r} R_2 + R_1 \to R_1 \\ R_2 \\ R_2 + R_3 \to R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & \tfrac{1}{2} & 2 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

No further simplifications can be performed and the resulting system is

$$\begin{aligned} x + \tfrac{1}{2}z &= 2 \\ y - 2z &= -1 \\ 0 &= 0 \end{aligned}$$

Since there are more variables than equations containing variables, we will have infinitely many solutions. To find the form of the solutions, solve both remaining equations in terms of *z*:

$$\begin{aligned} x &= 2 - \tfrac{1}{2}z \\ y &= -1 + 2z \end{aligned}$$

Our solutions will then have the form $(x, y, z) = (2 - \tfrac{1}{2}z, -1 + 2z, z)$, where z is any real number.

Inverse Method Writing the system in matrix form (**AX=B**) with

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 2.5 \\ 3 & -1 & 3.5 \\ -2 & 1 & -3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 3 \\ 7 \\ -5 \end{bmatrix}, \quad \text{and } \mathbf{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

one can try and solve for the variables, **X**, by finding the product $\mathbf{A}^{-1} \cdot \mathbf{B}$, if $\mathbf{A}^{-1}$ exists. In this case $\mathbf{A}^{-1}$ does not exist which tells us that this system either has no solution OR infinitely many solutions. To determine which case we have, we must solve the system by Gauss-Jordan elimination, as described previously.