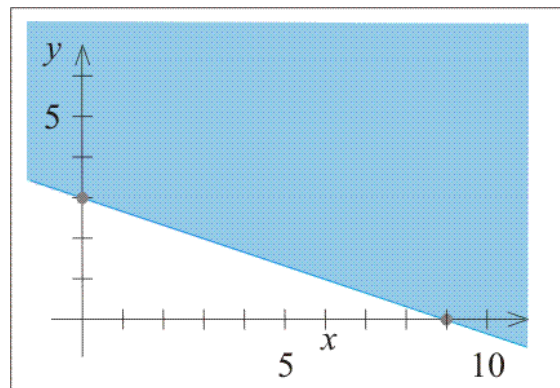
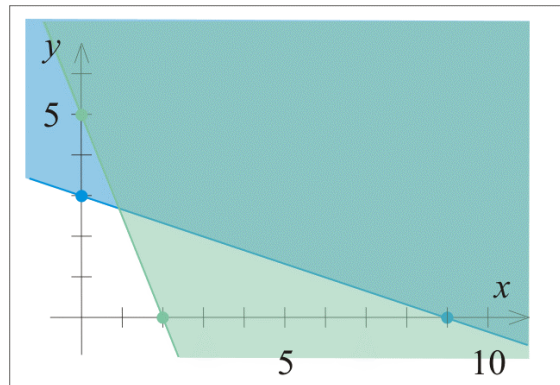


SOLUTIONS - CHAPTER 4 EXERCISES

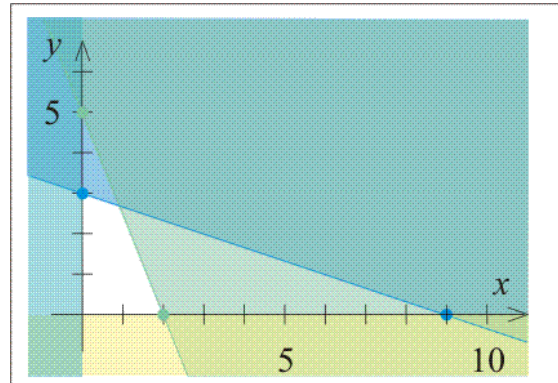
- 4.1 First, graph the equality, $x + 3y = 9$. It has an x -intercept of $(9,0)$ and a y -intercept of $(0,3)$. Test a point not on the line - $(0,0)$ is easy to calculate with and fits the requirement. Plugging $(0,0)$ into the inequality gives $0 + 3(0) \leq 9 \Rightarrow 0 \leq 9$, which is a true statement. This means that every point on the side of the line containing $(0,0)$ makes the inequality true and is, therefore, part of our feasible region. We will shade the false region to illustrate this.



Continue in the same manner for $5x + 2y \leq 10$. $5x + 2y = 10$ has an x -intercept of $(2,0)$ and a y -intercept of $(0,5)$. Testing $(0,0)$ gives us $5(0) + 2(0) \leq 10 \Rightarrow 0 \leq 10$, which is a true statement. Therefore, we shade the side of the line not containing $(0,0)$.



The constraints $x \geq 0$ and $y \geq 0$ restrict the solution to the first quadrant, so our feasible region is the following:

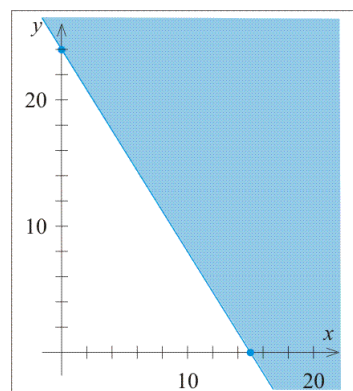


4.3 Using the information found in Exercises 4.1 and 4.2, construct a table containing the corner points and evaluate the objective function, $P = 3x - 2y$, at each of these points.

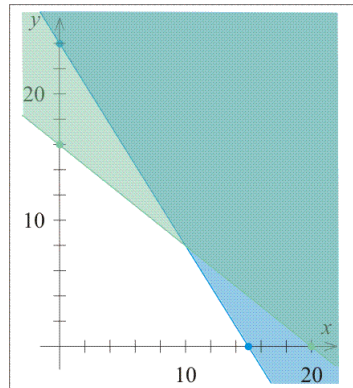
Corner	$P = 3x - 2y$	
(0,0)	$P = 3(0) - 2(0) = 0$	
(0,3)	$P = 3(0) - 2(3) = -6$	Minimum
(2,0)	$P = 3(2) - 2(0) = 6$	Maximum
$(\frac{12}{13}, \frac{35}{13})$	$P = 3(\frac{12}{13}) - 2(\frac{35}{13}) = -\frac{34}{13} \approx -2.62$	

The maximum value is 6 at (2,0) and the minimum value is -6 at (0,3).

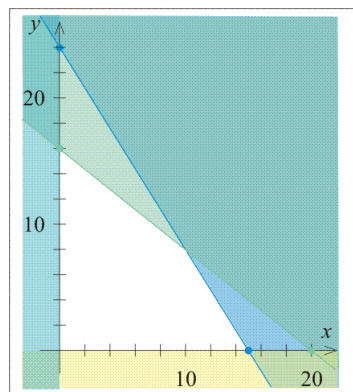
4.5 First, let x = the number of rocking chairs made and let y = the number of lounging chairs made. Next, find the feasible region formed by the constraints. Graph the equality $16x + 10y = 240$. It has an x -intercept of (15,0) and a y -intercept of (0,24). Testing a point, (0,0), which is not on the line gives $16(0) + 10(0) \leq 240$, which is a true statement and thus the side not containing the point (0,0) is shaded.



Graph the equality $2x + 2.5y = 40$. It has an x -intercept of $(20,0)$ and a y -intercept of $(0,16)$. Testing $(0,0)$ gives $2(0) + 2.5(0) \leq 40$, which is a true statement and again the side not containing $(0,0)$ is shaded.



The constraints $x \geq 0$ and $y \geq 0$ restrict the solution to the first quadrant and thus our feasible region, S , is shown in the following.



Now, we need to find the corner points. Three are on the axes: $(0,0)$, $(0,16)$, and $(15,0)$. The final corner is at the intersection of the lines $16x + 10y = 240$ and $2x + 2.5y = 40$. Using the techniques discussed in the solutions to the Chapter 1 Exercises, the intersection point is $(10,8)$.

Construct a table containing the corner points and evaluate the objective function at each of these points.

Corner	$R = 100x + 160y$	
$(0,0)$	$R = 100(0) + 160(0) = 0$	
$(0,16)$	$R = 100(0) + 160(16) = 2560$	Maximum
$(15,0)$	$R = 100(15) + 160(0) = 1500$	
$(10,8)$	$R = 100(10) + 160(8) = 2280$	

Therefore, the maximum revenue is \$2560 when they make 0 rocking chairs and 16 lounging chairs.

4.7 Using the initial tableau and pivot element found in the solution to Exercise 4.6, we have the following result:

$$\left[\begin{array}{cccccc|c} x & y & z & u & v & w & P \\ 1 & 3 & \langle 5 \rangle & 1 & 0 & 0 & 15 \\ 6 & 1 & 2 & 0 & 1 & 0 & 18 \\ 4 & 2 & 1 & 0 & 0 & 1 & 16 \\ \hline -4 & -1 & -5 & 0 & 0 & 0 & 1 & 0 \end{array} \right] \Rightarrow \left[\begin{array}{cccccc|c} x & y & z & u & v & w & P \\ \frac{1}{5} & \frac{3}{5} & 1 & \frac{1}{5} & 0 & 0 & 3 \\ \langle \frac{28}{5} \rangle & -\frac{1}{5} & 0 & -\frac{2}{5} & 1 & 0 & 12 \\ \frac{19}{5} & \frac{7}{5} & 0 & -\frac{1}{5} & 0 & 1 & 13 \\ \hline -3 & 2 & 0 & 1 & 0 & 0 & 1 & 15 \end{array} \right] \begin{array}{l} \text{Ratios} \\ 3 / (\frac{1}{5}) = 15 \\ 12 / (\frac{28}{5}) = \frac{15}{7} \approx 2.1 \\ 13 / (\frac{19}{5}) = \frac{65}{19} \approx 3.4 \end{array}$$

Since a negative element still lies in the bottom row, another pivot is required. The pivot column will be Column 1 and the pivot row will be Row 2. Thus, the pivot element is $\frac{28}{5}$, as indicated above.

After performing the indicated pivot, we have the following result:

$$\left[\begin{array}{cccccc|c} x & y & z & u & v & w & P \\ 0 & \frac{17}{28} & 1 & \frac{3}{14} & -\frac{1}{28} & 0 & \frac{18}{7} \\ 1 & -\frac{1}{28} & 0 & -\frac{1}{14} & \frac{5}{28} & 0 & \frac{15}{7} \\ 0 & \frac{43}{28} & 0 & \frac{1}{14} & -\frac{19}{28} & 1 & \frac{34}{7} \\ \hline 0 & \frac{53}{28} & 0 & \frac{11}{14} & \frac{15}{28} & 0 & 1 & \frac{150}{7} \end{array} \right]$$

Since there are no negative elements in the bottom row, we have reached an optimal solution. From the bottom row we can find that $P = \frac{150}{7}$. Thus, the maximum value of the objective function is $\frac{150}{7}$.

4.9 First, rewrite the inequalities as equalities with slack variables and rewrite the objective function:

$$\begin{aligned} 2x - y + 3z + u &= 24 \\ x + 5y + \quad v &= 15 \\ 3x + \quad 2z + w &= 18 \\ -x - 2y - 4z + P &= 0 \end{aligned}$$

Next, place this system into an augmented matrix (adding a horizontal line to separate the system coefficients from the objective function coefficients):

$$\left[\begin{array}{cccccc|c} x & y & z & u & v & w & P & \mathbf{Ratios} \\ 2 & -1 & \langle 3 \rangle & 1 & 0 & 0 & 0 & 24 \quad \frac{24}{3} = 8 \\ 1 & 5 & 0 & 0 & 1 & 0 & 0 & 15 \quad \text{No ratio} \\ 3 & 0 & 2 & 0 & 0 & 1 & 0 & 18 \quad \frac{18}{2} = 9 \\ \hline -1 & -2 & -4 & 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

The most negative element in the bottom row is -4 , indicating pivot Column 3. The smallest positive ratio is 8, indicating pivot Row 1. Thus, the pivot element is the 3 in Row 1, Column 3, as indicated above.