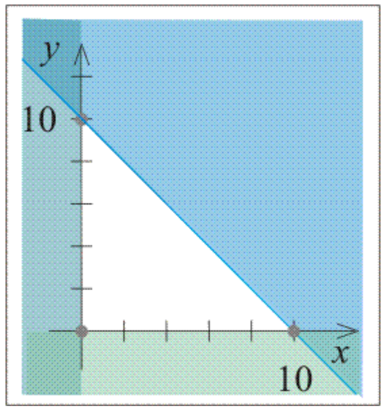
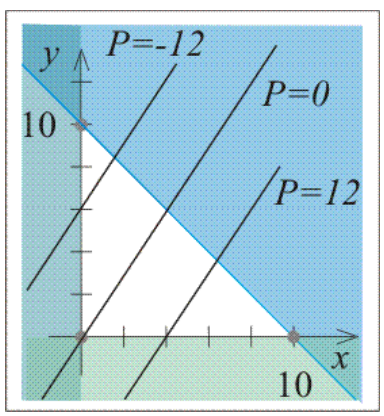
SOLUTIONS - CHAPTER 4 QUIZ

4.1 Graph the equality, $x + y = 10$. It has an x-intercept of $(10,0)$ and a y-intercept of $(0,10)$. Testing a point not on the line, say $(0,0)$, gives $0+0 \le 10 \Rightarrow 0 \le 10$, which is a true statement. This means that every point on the side of the line containing $(0,0)$ makes the inequality true and is, therefore, part of our feasible region. We will shade the false region to illustrate this. The constraints $x \ge 0$ and $y \ge 0$ restrict our solution to the first quadrant.



Thus, the feasible region has three corners on the axes: $(0,0), (0,10)$, and $(10,0)$.

4.2



The objective function is increasing in the direction of increasing x and decreasing y.

4.3 Using the feasible region found in the solution to Question 4.1, we can see the corners are at $(0,0), (0,10)$, and $(10,0)$.

4.4

Corner	***P* = 3*x* – 2*y***	
(0,0)	$P = 3(0) - 2(0) = 0$	
(0,10)	$P = 3(0) - 2(10) = -20$	
(10,0)	$P = 3(10) - 2(0) = 30$	**Maximum**

The maximum value of P occurs at $(10,0)$.

4.5 The maximum occurs at $(10,0)$ which is the intersection of the line $y=0$ and the line $x+y=10$, as seen from the graph of the feasible region in the solution to Question 4.1.

4.6 First, rewrite the inequalities as equalities with slack variables and rewrite the objective function:

$$\begin{aligned} 3x+y+2z+u&=9 \\ 2x+3y+z+v&=8 \\ x+2y+3z+w&=7 \\ -20x-12y-18z+P&=0 \end{aligned}$$

Next, place this system into an augmented matrix (adding a horizontal line to separate the system coefficients from the objective function coefficients) to form the initial tableau:

$$\left[\begin{array}{ccccccc|c} x & y & z & u & v & w & P & \\ 3 & 1 & 2 & 1 & 0 & 0 & 0 & 9 \\ 2 & 3 & 1 & 0 & 1 & 0 & 0 & 8 \\ 1 & 2 & 3 & 0 & 0 & 1 & 0 & 7 \\ \hline -20 & -12 & -18 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

4.7 There are 7 variables (x,y,z,u,v,w,P) as seen in the solution to Question 4.6.

4.8 Using the initial tableau from the solution to Question 4.6, the most negative element in the bottom row is -20, indicating pivot Column 1. The smallest positive ratio is 3, indicating pivot Row 1. Thus, the first pivot element is the 3 in Row 1, Column 1.

$$\left[\begin{array}{ccccccc|c} x & y & z & u & v & w & P & \\ \langle 3 \rangle & 1 & 2 & 1 & 0 & 0 & 0 & 9 \\ 2 & 3 & 1 & 0 & 1 & 0 & 0 & 8 \\ 1 & 2 & 3 & 0 & 0 & 1 & 0 & 7 \\ \hline -20 & -12 & -18 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} \textbf{Ratios} \\ 9/3=3 \\ 8/2=4 \\ 7/1=7 \\ \\ \end{array}$$

The result after pivoting is:

$$\left[\begin{array}{ccccccc|c} x & y & z & u & v & w & P & \\ 1 & 1/3 & 2/3 & 1/3 & 0 & 0 & 0 & 3 \\ 0 & 7/3 & -1/3 & 2/3 & 1 & 0 & 0 & 2 \\ 0 & 5/3 & 7/3 & -1/3 & 0 & 1 & 0 & 4 \\ \hline 0 & -16/3 & -14/3 & 20/3 & 0 & 0 & 1 & 60 \end{array}\right]$$

Using the bottom row, the value of the objective function at this point is 60.

4.9 Using the tableau found in the solution to Question 4.8, the most negative element in the bottom row is $-16/3$, indicating pivot Column 2. The smallest positive ratio is $6/7$, indicating pivot Row 2. Thus, the next pivot element is the $7/3$ in Row 2, Column 2.

x	y	z	u	v	w	P		**Ratios**
1	$1/3$	$2/3$	$1/3$	0	0	0	3	$3/(1/3)=9$
0	$\langle 7/3 \rangle$	$-1/3$	$2/3$	1	0	0	2	$2/(7/3)=6/7$
0	$5/3$	$7/3$	$-1/3$	0	1	0	4	$4/(5/3)=12/5$
0	$-16/3$	$-14/3$	$20/3$	0	0	1	60	

The result after pivoting is:

x	y	z	u	v	w	P	
1	0	$5/7$	$3/7$	$-1/7$	0	0	$19/7$
0	1	$-1/7$	$-2/7$	$3/7$	0	0	$6/7$
0	0	$18/7$	$1/7$	$-5/7$	1	0	$18/7$
0	0	$-38/7$	$36/7$	$16/7$	0	1	$452/7$

Using the bottom row, the value of the objective function at this point is $452/7 \approx 64.6$.

4.10 Using the tableau found in the solution to Question 4.9, the only remaining negative element in the bottom row is $-38/7$, indicating pivot Column 3. The smallest positive ratio is 1, indicating pivot Row 3. Thus, the pivot element is $18/7$ in Row 3, Column 3.

x	y	z	u	v	w	P		**Ratios**
1	0	$5/7$	$3/7$	$-1/7$	0	0	$19/7$	$(19/7)/(5/7)=19/5$
0	1	$-1/7$	$-2/7$	$3/7$	0	0	$6/7$	No ratio
0	0	$\langle 18/7 \rangle$	$1/7$	$-5/7$	1	0	$18/7$	$(18/7)/(18/7)=1$
0	0	$-38/7$	$36/7$	$16/7$	0	1	$452/7$	

The result after pivoting is:

x	y	z	u	v	w	P	
1	0	0	$7/18$	$1/18$	$-5/18$	0	2
0	1	0	$-5/18$	$7/18$	$1/18$	0	1
0	0	1	$1/18$	$-5/18$	$7/18$	0	1
0	0	0	$49/9$	$7/9$	$19/9$	1	70

Since there are no negative elements remaining in the bottom row, we find that the maximum value of the objective function is 70.