SOLUTIONS - CHAPTER 5 QUIZ

5.1 a. False $\rightarrow \varnothing$ is not an *element* in U.

b. True $\rightarrow A \cup B = \{0,2,4\} \cup \{1,2,3,4,5\} = \{0,1,2,3,4,5\} = U$.

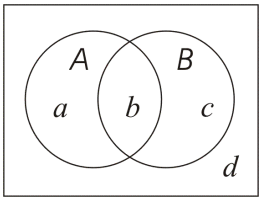
c. True $\rightarrow$ Every element in C is an element in B and the sets are not equal.

d. False $\rightarrow C^c \cap A = \{0,1,5\} \cap \{0,2,4\} = \{0\} \neq \varnothing$.

5.2 $A = \{2,4,6\}$ and $A^c = \{0,1,3,5,7,8,9\}$.

$n(A) =$ the number of elements in $A = 3$; $n\left(A^c\right) =$ the number of elements in U, not in $A = 7$.

5.3 Draw and label each section of a Venn diagram with sets A and B:



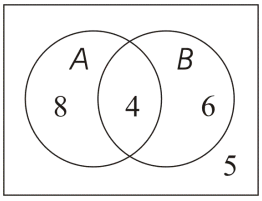
We are given the following information:

$$n(A) = a + b = 12$$
$$n(B) = b + c = 10$$
$$n(A \cup B) = a + b + c = 18$$
$$n\left(A^c \cap B^c\right) = d = 5$$

Using substitution we find:

$$12 + c = 18 \Rightarrow c = 6$$
$$b + 6 = 10 \Rightarrow b = 4$$
$$a + 4 = 12 \Rightarrow a = 8$$

Therefore, we have



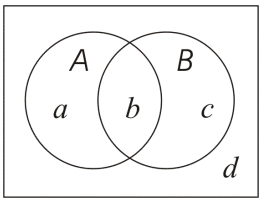
So,

$n(A \cap B)$ = the number of elements in both A and $B = b = 4$

$n\left(A^c \cap B\right)$ = the number of elements in B, but not in $A = c = 6$

$n\left(A \cup B^c\right)$ = the number of elements in A, plus the number of elements not in $B = a + b + d = 17$

5.4 Draw and label each section of a Venn diagram with sets A and B:



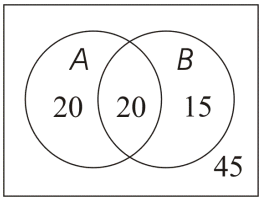
We are given the following information:

$$n(A) = a + b = 40$$
$$n(B) = b + c = 35$$
$$n(A \cup B) = a + b + c = 55$$
$$n(U) = a + b + c + d = 100$$

Using substitution we find:

$$40 + c = 55 \Rightarrow c = 15$$
$$b + 15 = 35 \Rightarrow b = 20$$
$$a + 20 = 40 \Rightarrow a = 20$$
$$20 + 20 + 15 + d = 100 \Rightarrow d = 45$$

Therefore, we have



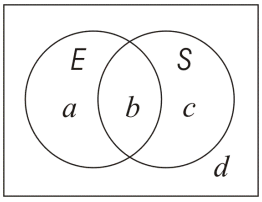
So,

$n(A \cap B)$ = the number of elements in both A and $B = b = 20$

$n\left(A^c \cap B\right)$ = the number of elements in B, but not in $A = c = 15$

$n\left(A \cup B^c\right)$ = the number of elements in A, plus the number of elements not in $B = a + b + d = 85$

5.5 Let U = the set of shoppers surveyed, E = the set of shoppers who bought eggs, and S = the set of shoppers who bought steak. Draw and label each section of a Venn diagram containing the defined sets:



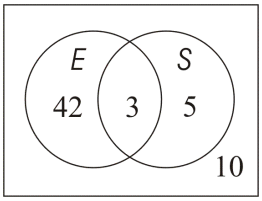
We are given

$$n(U) = a + b + c + d = 60$$
$$n(E) = a + b = 45$$
$$n(S) = b + c = 8$$
$$n\left(E^c \cap S^c\right) = d = 10$$

Using substitution we find:

$$45 + + c + 10 = 60 \Rightarrow c = 5$$
$$b + 5 = 8 \Rightarrow b = 3$$
$$a + 3 = 45 \Rightarrow a = 42$$

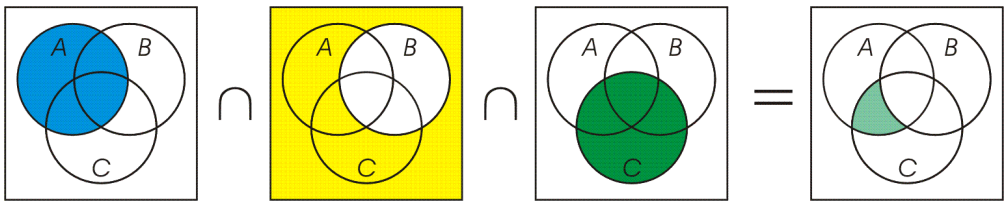
Therefore, we have



Reading from our Venn diagram we find the following answers to the questions asked.

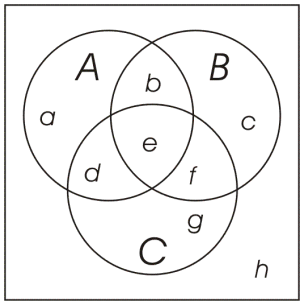
a. $42 + 3 + 5 = 50$
b. 5
c. 3

5.6



5.7 INSERT VENN

5.8 Draw and label each section of a Venn diagram containing the given sets, A, B, and C:



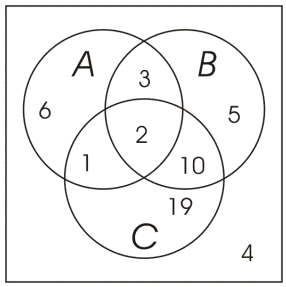
We are given the following information:

$$n(U) = a+b+c+d+e+f+g+h = 50$$
$$n(A \cap B \cap C) = e = 2$$
$$n(A \cap B) = b+e = 5$$
$$n(A) = a+b+d+e = 12$$
$$n(A \cap B^c \cap C) = d = 1$$
$$n\left((B \cup C)^c\right) = a+h = 10$$
$$n\left(\left(A \cap B^c \cap C\right) \cup \left(A^c \cap B \cap C\right)\right) = d+f = 11$$
$$n\left(A^c \cap B \cap C^c\right) = c = 5$$

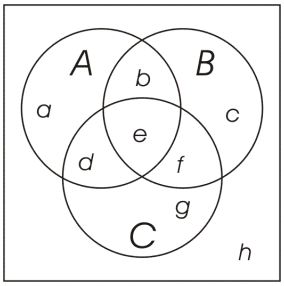
Using substitution we find:

$$a+b+c+d+e+f+g+h = 50 \Rightarrow 6+3+5+1+2+10+g+4 = 50 \Rightarrow g = 19$$
$$e = 2$$
$$b+e = 5 \Rightarrow b+2 = 5 \Rightarrow b = 3$$
$$a+b+d+e = 12 \Rightarrow a+3+1+2 = 12 \Rightarrow a = 6$$
$$d = 1$$
$$a+h = 10 \Rightarrow 6+h = 10 \Rightarrow h = 4$$
$$d+f = 11 \Rightarrow 1+f = 11 \Rightarrow f = 10$$
$$c = 5$$

Therefore, our Venn diagram will look like:



5.9 Draw and label each section of a Venn diagram containing the given sets, A, B, and C:



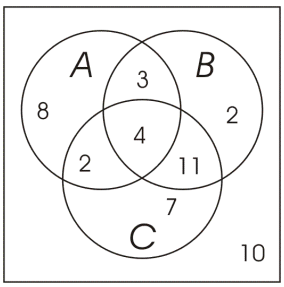
We are given the following information:

$$n(B) = b + c + e + f = 20$$
$$n(A \cap B) = b + e = 7$$
$$n(A \cap B \cap C) = e = 4$$
$$n(A \cap B^c \cap C^c) = a = 8$$
$$n(A^c \cap B^c \cap C^c) = h = 10$$
$$n(A \cup B \cup C) = a + b + c + d + e + f + g = 37$$
$$n\left(\left(A \cap B \cap C^c\right) \cup \left(A^c \cap B \cap C\right)\right) = b + f = 14$$
$$n\left(\left(A \cup B\right)^c\right) = g + h = 17$$

Using substitution we find:

$$b + c + e + f = 20 \Rightarrow 3 + c + 4 + 11 = 20 \Rightarrow c = 2$$
$$b + e = 7 \Rightarrow b + 4 = 7 \Rightarrow b = 3$$
$$e = 4$$
$$a = 8$$
$$h = 10$$
$$a + b + c + d + e + f + g = 37 \Rightarrow 8 + (20) + d + 7 = 37 \Rightarrow d = 2$$
$$b + f = 14 \Rightarrow 3 + f = 14 \Rightarrow f = 11$$
$$g + h = 17 \Rightarrow g + 10 = 17 \Rightarrow g = 7$$

Therefore, our Venn diagram will look like:



Using this Venn diagram we find the answers to the questions asked.

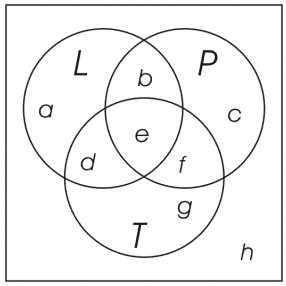
a. $n(A \cup B) = a + b + c + d + e + f = 8 + 3 + 2 + 2 + 4 + 11 = 30$

b. $n(C^c) = a + b + c + h = 8 + 3 + 2 + 10 = 23$

c. $n(U) = a + b + c + d + e + f + g + h = 8 + 3 + 2 + 2 + 4 + 11 + 7 + 10 = 47$

5.10 Let U = the set of children surveyed, L = the set of children who likes lettuce, P = the set of children who likes pickles, and T = the set of children who likes tomatoes.

Draw and label each section of a Venn diagram containing the given sets, L, P, and T:



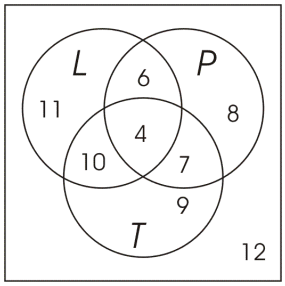
We are given the following information:

$$\begin{aligned} c &= 8 \\ e &= 4 \\ b + d + f &= 23 \\ f &= 7 \\ b + e &= 10 \\ d + e + f + g &= 30 \\ a + b &= 17 \\ c + h &= 20 \end{aligned}$$

Using substitution we find:

$$\begin{aligned} c &= 8 \\ e &= 4 \\ b + d + f &= 23 \Rightarrow 6 + d + 7 = 23 \Rightarrow d = 10 \\ f &= 7 \\ b + e &= 10 \Rightarrow b + 4 = 10 \Rightarrow b = 6 \\ d + e + f + g &= 30 \Rightarrow 10 + 4 + 7 + g = 30 \Rightarrow g = 9 \\ a + b &= 17 \Rightarrow a + 6 = 17 \Rightarrow a = 11 \\ c + h &= 20 \Rightarrow 8 + h = 20 \Rightarrow h = 12 \end{aligned}$$

Therefore, we have



Using this information, we can answer the given questions.

a. $a+b+c+d+e+f+g+h=67$
b. $b+c+e+f=25$
c. $b+d+e+f=27$