SOLUTIONS - CHAPTER 6 EXERCISES

6.1 To compose a mini-sundae, there are three tasks to complete: choose an ice-cream flavor, choose a sauce, and choose a topping. Using the multiplication principle:

$$\underset{\text{Flavor}}{\underline{\qquad}} \times \underset{\text{Sauce}}{\underline{\qquad}} \times \underset{\text{Topping}}{\underline{\qquad}} \rightarrow \underset{\text{Flavor}}{\underline{27}} \times \underset{\text{Sauce}}{\underline{6}} \times \underset{\text{Topping}}{\underline{10}} = 1620$$

There are 1620 different mini-sundaes possible, assuming one ice-cream flavor, sauce, and topping are used.

6.3 Using the multiplication principle, we have 18 tasks to complete.

Method 1
There are 18 choices for who sits in the first seat. Alternating adults and children leaves 9 people to choose from to sit in the second seat, 8 for the next, etc. Therefore, there are

$$18 \cdot 9 \cdot 8 \cdot 8 \cdot 7 \cdot 7 \cdot 6 \cdot 6 \cdot 5 \cdot 5 \cdot 4 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1 \approx 2.6336 \times 10^{11}$$

ways to alternately seat the adults and the children.

Method 2
There are two ways to seat the people while alternating adults and children – you either start with an adult or you start with a child. Using the multiplication principle, there are

$$\underset{A}{\underline{9}} \cdot \underset{C}{\underline{9}} \cdot \underset{A}{\underline{8}} \cdot \underset{C}{\underline{8}} \cdot \underset{A}{\underline{7}} \cdot \underset{C}{\underline{7}} \cdot \underset{A}{\underline{6}} \cdot \underset{C}{\underline{6}} \cdot \underset{A}{\underline{5}} \cdot \underset{C}{\underline{5}} \cdot \underset{A}{\underline{4}} \cdot \underset{C}{\underline{4}} \cdot \underset{A}{\underline{3}} \cdot \underset{C}{\underline{3}} \cdot \underset{A}{\underline{2}} \cdot \underset{C}{\underline{2}} \cdot \underset{A}{\underline{1}} \cdot \underset{C}{\underline{1}} = 9!9!$$

ways to start the seating with an adult.

Similarly, there are 9!9! ways to start the seating with a child.

Thus, there are a total of $9! \cdot 9! + 9! \cdot 9! = 2 \cdot 9! \cdot 9!$ ways to arrange the group.

6.5 There are two tasks to complete in order to obtain a single outcome: choose a letter and toss a coin. Using the multiplication principle,

$$\underset{\text{Draw Letter}}{\underline{\qquad\quad}} \times \underset{\text{Toss Coin}}{\underline{\qquad\quad}} \rightarrow \underset{\text{Draw Letter}}{\underline{7}} \times \underset{\text{Toss Coin}}{\underline{2}} = 14$$

There are 14 outcomes that are equally likely.

6.7 The sample space for this experiment is $S = \{\text{w}, \text{x}, \text{y}, \text{z}\}$. The events are all of the possible subsets of *S*. These include

$$\varnothing, \{\text{w}\}, \{\text{x}\}, \{\text{y}\}, \{\text{z}\}, \{\text{w, x}\}, \{\text{w, y}\}, \{\text{w, z}\}, \{\text{x, y}\}, \{\text{x, z}\}, \{\text{y, z}\},$$
$$\{\text{w, x, y}\}, \{\text{w, x, z}\}, \{\text{w, y, z}\}, \{\text{x, y, z}\}, \text{ and } S = \{\text{w, x, y, z}\}.$$

So, there are 16 possible events.

6.9 There are $C(9,1)=9$ ways to choose 1 of the freshmen, $C(6,1)=6$ ways to choose 1 of the sophomores, $C(3,1)=3$ ways to choose one of the juniors, and $C(1,1)=1$ way to choose 1 of the seniors. Thus, using the multiplication principle there are $9\cdot 6\cdot 3\cdot 1=162$ ways to choose a group with exactly 1 freshman, 1 sophomore, 1 junior, and 1 senior.

The total number of ways to choose a four-person group is to choose 4 of the 19 students, which can happen in $C(19,4)=3876$ ways.

Therefore, the *probability* that a four-person group has exactly one student of each classification is equal to

$$\frac{\text{Number of four-person groups with exactly 1 student of each classification}}{\text{Total number of four-person groups}}=\frac{162}{3876}=\frac{27}{646}\approx 0.0418$$