SOLUTIONS - CHAPTER 6 QUIZ

6.1 To order a special plate, there are three tasks to complete: choose an entree, choose a salad, and choose a side dish. Using the multiplication principle:

$$\underset{\text{Entree}}{\underline{\quad\quad}} \times \underset{\text{Salad}}{\underline{\quad\quad}} \times \underset{\text{Side Dish}}{\underline{\quad\quad}} \rightarrow \underset{\text{Entree}}{\underline{\ 6\ }} \times \underset{\text{Salad}}{\underline{\ 8\ }} \times \underset{\text{Side Dish}}{\underline{\ 3\ }} = 144$$

There are 144 different special plates possible.

6.2 This is a permutation since we are arranging items in a finite set in a specific order. Thus, there are $P(7,7) = 7! = 5040$ different arrangements for the seven bottles.

6.3 Using the multiplication principle, we have 10 tasks to complete.

Method 1
There are 10 choices for which book can be placed on the shelf first. Alternating the colors leaves 5 books to choose from to be placed second, 4 for the next, etc. Therefore, there are

$$10 \cdot 5 \cdot 4 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 = 28{,}800$$

ways to arrange the books, alternating colors.

Method 2
There are two ways to set the books on the shelf, while alternating colors – you either start with green book or you start with a yellow book. Using the multiplication principle, there are

$$\underset{\text{G}}{\underline{5}} \cdot \underset{\text{Y}}{\underline{5}} \cdot \underset{\text{G}}{\underline{4}} \cdot \underset{\text{Y}}{\underline{4}} \cdot \underset{\text{G}}{\underline{3}} \cdot \underset{\text{Y}}{\underline{3}} \cdot \underset{\text{G}}{\underline{2}} \cdot \underset{\text{Y}}{\underline{2}} \cdot \underset{\text{G}}{\underline{1}} \cdot \underset{\text{Y}}{\underline{1}} = 14400$$

ways to start the arrangement with a green book.

Similarly, there are 14400 ways to start the arrangement with a yellow book.

Thus, there are a total of $14400 + 14400 = 28{,}800$ ways to arrange the books in this manner.

6.4 The word "workbook" has 8 letters with repetition of 3 o's and 2 k's. There are 8! ways to arrange all of the letters. There are 3! ways to arrange the o's and 2! ways to arrange the k's. Thus, the number of distinguishable "words" that can be created is

$$\frac{8!}{3! \cdot 2!} = 3360$$

6.5 There are two tasks to complete in order to obtain a single outcome: roll the six-sided die and roll the twelve-sided die. Using the multiplication principle,

$$\underset{\text{Six-Sided Die}}{\underline{\quad\quad\quad}} \times \underset{\text{Twelve-Sided Die}}{\underline{\quad\quad\quad}} \rightarrow \underset{\text{Six-Sided Die}}{\underline{\ 6\ }} \times \underset{\text{Twelve-Sided Die}}{\underline{\ 12\ }} = 72$$

There are 72 outcomes that are equally likely.

6.6 We can arrange the B's so there is a space between each:

_B_B_B_B_B_B_B_B_

Therefore, there are 9 places to place an A. Since order does not matter, there are $C(9,3) = 84$ ways to place the A's so they are not next to each other.

6.7 The sample space for this experiment is $S = \{\text{Blue, Red, Yellow}\} = \{\text{B, R, Y}\}$. The events are all of the possible subsets of *S*. These include

$$\varnothing, \{\text{B}\}, \{\text{R}\}, \{\text{Y}\}, \{\text{B, R}\}, \{\text{B, Y}\}, \{\text{R, Y}\}, \text{ and } S = \{\text{B, R, Y}\}.$$

So, there are 8 possible events.

6.8 A sample that has "at least 1 double yolk egg" means that the sample has exactly 1 double yolk egg or the sample has exactly 2 double yolk eggs.

To have exactly 1 double yolk egg means that the sample has 1 double yolk egg and 1 single yolk egg. This means we have to choose 1 of the 3 double yolk eggs and there are $C(3,1) = 3$ ways to do so, and we have to choose 1 of the 9 single yolk eggs and there are $C(9,1) = 9$ ways to do so. Thus, in all, there are $3 \times 9 = 27$ ways to choose a sample with exactly 1 double yolk egg.

To have exactly 2 double yolk eggs we have to choose 2 of the 3 double yolk eggs and there are $C(3,2) = 3$ ways to do so. No further eggs need to be chosen for the sample, so no single yolk eggs can be chose in $C(9,0) = 1$ way. Thus, in all, there are $3 \times 1 = 3$ ways to choose a sample with exactly 2 double yolk eggs.

So, the total number of ways to choose a two-egg sample that has at least 1 double yolk egg is $27 + 3 = 30$.

The total number of ways to choose a two-egg sample is to choose 2 of the 12 eggs, which can happen in $C(12,2) = 66$ ways.

Therefore, the *probability* that a two-egg sample has at least 1 double yolk egg is equal to

$$\frac{\text{Number of two - egg samples with at least 1 double yolk egg}}{\text{Total number of two - egg samples}} = \frac{30}{66} = \frac{5}{11} \approx 0.4545$$

6.9 A sample of 3 marbles that has exactly one marble of each color means the sample will have 1 red, 1 white, and 1 blue marble. This means 1 of the 4 red marbles must be chosen, which can happen in $C(4,1) = 4$ ways, 1 of the 5 white marbles must be chosen, which can happen in $C(5,1) = 5$ ways, and 1 of the 3 blue marbles must be chosen, which can happen in $C(3,1) = 3$ ways. Thus, there are $4 \times 5 \times 3 = 60$ ways to choose a sample with exactly one marble of each color.

The total number of ways to choose a three-marble sample is to choose 3 of the 12 marbles, which can happen in $C(12,3) = 220$ ways.

Therefore, the *probability* that a three-marble sample has exactly one marble of each color is equal to

$$\frac{\text{Number of three - marble samples with exactly one marble of each color}}{\text{Total number of three - marble samples}} = \frac{60}{220} = \frac{3}{11} \approx 0.2727$$

6.10 The sample space for rolling two fair six-sided dice are the following 36 equally likely outcomes:

1~1	2~1	3~1	4~1	5~1	6~1
1~2	2~2	3~2	4~2	5~2	6~2
1~3	2~3	3~3	4~3	5~3	6~3
1~4	2~4	3~4	4~4	5~4	6~4
1~5	2~5	3~5	4~5	5~5	6~5
1~6	2~6	3~6	4~6	5~6	6~6

We need to count the outcomes with sums of 8, outcomes where a 5 is showing, and outcomes where both of these occur since we are using the inclusive "or". There are a total of 14 outcomes (highlighted above) and thus the probability is $14/36 = 7/18 \approx 0.3889$.